

Printed Pages – 7

Roll No. :

328351(14)

B. E. (Third Semester) Examination, Nov.-Dec. 2021

(New Scheme)

(ET&T Branch)

MATHEMATICS-III

Time Allowed : Three hours

Maximum Marks : 80

Minimum Pass Marks : 28

Note : Attempt all questions. Part (a) is compulsory and attempt any two from (b), (c) and (d) in each unit.

Unit-I

1. (a) What is the first shifting property of Laplace transform?

2

[2]

(b) (i) Find $L \left\{ \frac{\cos at - \cos bt}{t} \right\}$

(ii) Evaluate $L \int_0^{\infty} \frac{e^{-at} - e^{-bt}}{t} dt$ 7

(c) Find the inverse Laplace transform of

$$f(s) = \log \frac{s^2 + 1}{s(s+1)} \quad 7$$

(d) Solve the following equation by transform method

$$(D^3 - 3D^2 + 3D - 1) y = t^2 e^t \text{ . given that}$$

$$y(0) = 1, y'(0) = 0, y''(0) = -2 \quad 7$$

Unit-II

2. (a) State Cauchy's integral. 2

(b) If $f(z)$ is a regular function of z , prove that :

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) |f(z)|^2 = 4 |f'(z)|^2 \quad 7$$

Or

[3]

Determine the analytic function $f(z) = u + iv$, if

$$u - v = \frac{\cos x + \sin x - e^{-y}}{2(\cos x - \cos by)} \text{ \& } f(\pi/2) = 0.$$

(c) Expand in the series the function,

$$f(z) = \frac{1}{(z-1)(z-2)} \text{ in the region.}$$

(i) $0 < |z| < 1$

(ii) $1 < |z| < 2$

(iii) $|z| > 2$

(iv) $0 < |z-1| < 1$ 7

(d) By Integrating around a unit circle,

$$\text{Evaluate : } \int_0^{2\pi} \frac{\cos 3\theta}{5 - 4 \cos \theta} d\theta \quad 7$$

Unit-III

3. (a) Define Karl Pearson's coefficient of correlation. 2

[4]

(b) Calculate the coefficient of correlation from the following data : 7

Marks in Physics	45	56	39	54	45	40	56	60	30	36
Marks in Maths	40	36	30	44	36	32	45	42	20	36

(c) In partially destroyed lab record of an analysis of correlation data, only the following results are available : variance of $x = 9$.

The line of regression of y on x and x on y are $4x - 5y + 33 = 0$, $20x - 9y = 10y$ respectively. 7

Calculate :

- The mean value of x and y
 - Standard deviation of y
 - The coefficient of correlations between x and y
- (d) A computer while calculating the correlation coefficient between two variate x and y from 25 pairs of observation. Obtain the following constants :
 $n = 25$, $\sum x = 125$, $\sum x^2 = 650$, $\sum y = 100$,
 $\sum y^2 = 960$, $\sum xy = 508$.

[5]

It was however, later discovered at the time of checking that he had copied down two pairs as

x	y
6	14
8	6

write the correct values ax .

x	y
8	12
6	8

Obtain the correct value of the correlation coefficient. 7

Unit-IV

4. (a) What is Rodrigue's formula. 2
- (b) Solve in series the equation :

$$4x \frac{d^2y}{dx^2} + 2(1-x) \frac{dy}{dx} - y = 0, \quad 7$$

(c) Prove that

$$\int_{-1}^1 x^m P_n(x) dx = 0$$

where m, n are positive integers and $m < n$. 7

[6]

- (d) Express $f(x) = x^4 + 2x^3 + 2x^2 - x - 2$ in terms of Legendre polynomials.

7

Unit-V

5. (a) Solve :

$$yzp + zxq = xy.$$

2

- (b) Solve :

$$y^2p - xyq = x(z - 2y).$$

7

- (c) Using the method of separation of variables, solve :

$$\frac{\partial u}{\partial x} = 2 \frac{\partial u}{\partial t} + u$$

$$\text{Given } u(x, 0) = 6e^{-3x}.$$

7

- (d) A tightly stretched string of length l with fixed ends is initially in equilibrium position. It is set vibrating by giving each point a velocity $v_0 \sin 3\pi x/l$. Find the displacement $y(x, t)$.

7

Or

[7]

A string is stretched between the fixed points $(0, 1)$ and $(1, 0)$ released from rest from the position $U = A \sin \pi x$, find the formula for its subsequent displacement $u(x, t)$.